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ORIGINAL ARTICLE

Decentralized low-order ADRC design for MIMO system with unknown order and relative degree

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Abstract
Many industrial processes are multi-input multi-output (MIMO) systems, in which the order and/or the relative degree are uncertain, and the parameters cannot be obtained accurately. In this paper, for such MIMO systems, the decentralized low-order active disturbance rejection control (ADRC) scheme is designed with tuning method, and the whole design procedure is verified on the plants with/without the time delay. A necessary condition is derived based on the definition of numerator's zero-order coefficient (NZC) matrix for the closed-loop stability. It is proved that the low-order ADRC scheme can reject the interaction disturbance and guarantee the closed-loop stability for the open-loop stable MIMO plants. This design method is capable of guaranteeing the integrity for the open-loop stable plant with diagonally dominant NZC matrix, which has also discussed in the simulations. Several numerical new methods have given to show the further capability of the ADRC scheme to obtain better performances for the systems with/without time delay.

Keywords ADRC · MIMO system · Integrity · Time delay · Parameter tuning

1 Introduction

Most industrial processes have more than one variable in nature, and this fact will increase in future [1]. The decentralized control structure shows advantages to the control engineers, since it can be easily understood and implemented in controlling the multi-input multi-output (MIMO) process [2, 3]. The decentralized PID control is one of the most common control schemes among all kinds of the decentralized schemes (see [4–7] for examples).

The traditional decentralized approaches can work properly when the interactions in different channels of the process are modest [1]. Nevertheless, the MIMO systems present unknown and complicated couplings among the measurements and control signals, with the uncertain orders and relative degrees. Hence, many scholars are interested in the new techniques for controlling the MIMO system.

The active disturbance rejection control (ADRC) technique regards the coupling interaction as a part of the uncertainty in one single channel. The ADRC scheme can observe and reject the total uncertainty in every channel, which includes both the internal (parameter or unmodeled dynamics) uncertainties and the external uncertainties (disturbance) [8–11]. This idea is very suitable for the decentralized controller design. Hence, ADRC has been employed for the MIMO systems in many experiment and simulation researches. The ADRC is designed for a two-input-two-output process with time delay in [12]. In [13], two nonlinear ADRCs are designed for the boiler master fuel control and the turbine load control of the power plant unit coordinated system. The researches in [14, 15] show that the linear ADRC can achieve high performance and good robustness in practical application. And more researches can be found in [3, 13, 16–18].

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Recently, the theoretical analysis is made for the MIMO system controlled by the ADRC scheme. The literatures [19, 20] concentrate on the linear form of ADRC. In [19], the closed loop is proved semi-global stable for a class of lower triangular MIMO plants, which contains uncertain nonlinear time-varying dynamics and discontinuous external disturbances. In [20], the capability of ADRC is analyzed for several kinds of nonlinear MIMO systems whose orders and relative degrees for all channels are equal to the same number. The nonlinear ADRC for the MIMO system is discussed in the [21, 22]. In [21], the convergence is investigated for the nonlinear extended state observer (ESO), which is an important part of the ADRC. In addition, a sufficient condition is given in [22] for ADRC stabilizing the nonlinear MIMO plant, of which the relative degree equals the order for every channel. In the former researches, the orders or the relative degrees of all the channels in the plant have assumed to know and utilize to determine the order of ADRC scheme.

However, for many industrial plants, it is costly or difficult to obtain the precise information of their models, including the orders and the relative degrees [23]. Even if the model is exactly known, the order or the relative degree may be a big number. For example, the order of the accurate model is very high for a multi machine of power system, since each synchronous generator includes the prime mover, the governor, and the exciter, and each component contributes to the final order of the whole model [24]. In fact, the high order is from the complexity of the structure or dynamics in the equipment, and more such examples can be seen in [25–27] and the references therein. According to the existing theoretical results, ADRC schemes with high-order observers carry more parameters than the low-order ADRC schemes, which are more difficult for the engineers to design and adjust. Hence, many ADRC design methods in application (such as [3, 28–30]) do not require accurate information of the order or the relative degree, and usually employ the ADRC schemes with low-order observers.

This paper adopts the engineering experiences in designing low-order ADRC scheme for uncertain MIMO plants. The presented ADRC design does not need the information of the orders or the relative degrees. The stability and the high integrity of the closed loop are discussed in theory. The sufficient conditions are given for the ADRC closed-loop stability and the integrity, and a necessary result is established for the closed-loop stability. The design method and the theoretical results are all validated in the simulations.

The rest of the paper is arranged as follows: in Section 2, the problem is formulated by building the controller structure and making the primary analysis, and the plant models are given for later use, which are adopted from [3, 31]. Section 3 is on the necessary condition for the closed-loop stability. In Section 4, the tuning method for the low-order ADRC scheme is illustrated in the flowchart; the sufficient condition for the closed-loop stability is established as the theoretical support; and the method is verified in the simulations of the ADRC based the second-order ESO. Then, theory and simulations in Section 5 discuss the integrity of the ADRC closed loop. In Section 6, the decentralized ADRC scheme with the third-order ESO is illustrated based on the same design and tuning method. The last section is the conclusion.

2 Control design and preliminary analysis

Consider the MIMO LTI system with the following transfer-function model

$$\overrightarrow{Y}(s) = H(s)\overrightarrow{U}(s), \tag{1}$$

where $\overrightarrow{Y}(s) = [Y_1(s)\ \dots\ Y_m(s)]^T$ and $\overrightarrow{U}(s) = [U_1(s)\ \dots\ U_m(s)]^T$ are the Laplace transforms of the output vector $\overrightarrow{y}(t) = [y_1(t)\dots y_m(t)]^T$ and the input vector $\overrightarrow{u}(t) = [u_1(t)\ \dots\ u_m(t)]^T$, respectively. The transfer function is noted by $H(s) = H_D^{-1}(s)H_N(s)$, where

$$H_D(s) = \mathrm{diag}(h_{D1}, \dots,\ h_{Dm}(s)) = \begin{bmatrix} h_{D1}(s) & & \\ & \ddots & \\ & & h_{Dm}(s) \end{bmatrix}, \tag{2}$$

$h_{Di}(s) = \sum_{k=0}^{m_i} h_{Dik}s^k$ ($h_{Dim_i} = 1$ for $1 \le i \le m$), $H_N(s) = [b_{ij}h_{Nij}(s)]_{m\times m}$, $h_{Nij}(s) = \sum_{k=0}^{m_{ij}} h_{Nijk}s^k$ ($m_{ij} \le m_i$, $h_{Nijm_{ij}} \ne 0$, $h_{Nij0} = 1$, for $1 \le i \le m$, $1 \le j \le m$), and $H_D(s)$ and $H_N(s)$ are left coprime invertible polynomial matrices. Define the NZC (numerator's zero-order coefficient) matrix of the plant (1) as

$$B_P = (b_{ij})_{m\times m} = H_N(0),$$

Moreover, assume $\det B_P \ne 0$.

Remark 1 Since B_P is the zero-order coefficient matrix of the transfer function's numerator $H_N(s)$, it is called the NZC (numerator's zero-order coefficient) matrix for short in this work. When $m = 1$, the plant (1) becomes a SISO system, and the NZC matrix equals the numerator's zero-order coefficient (NZC) defined in the literatures [32, 33]. In fact, the NZC matrix for the MIMO system is a generalized form of the NZC for the SISO system.

The objective is to stabilize the plant (1) with the decentralized ADRC scheme. And the design procedure of the ADRC is shown in the time domain [9, 10, 34].

For the ith channel of the system (1), the time-domain form can be written as

$$y_i^{(m_i)} = -\sum_{k=0}^{m_i-1} h_{Dik}y_i^{(k)} + \sum_{j=1}^{m} b_{ij} \sum_{k'=0}^{m_{ij}} h_{Nijk'}u_j^{(k')}, \tag{3}$$

where $y^{(0)}=y$ and $u^{(0)}=u$; $y_i^{(k)}$ is the k-order derivative of $y_i(t)$ and $u_j^{(k')}$ is the k'-order derivative of $u_j(t)$. Due to the lack of the plant knowledge, the channel may be described by the dynamic system:

$$y_i^{(n_i)}=f_i+b_{Ei}u_i, \tag{4}$$

where $n_i \geq 1$ is the estimate of the order m_i; b_{Ei} is an estimate of b_{ii}; and f_i is called the total uncertainty (or total disturbance) in the ith channel [10]. Specifically,

$$f_i=-y_i^{(m_i)}-\sum_{k=0}^{m_i-1} h_{Dik} y_i^{(k)}+y_i^{(n_i)}+\sum_{j=1}^{m} b_{ij} \sum_{k'=0}^{m_{ij}} h_{Nijk'} u_j^{(k')}-b_{Ei}u_i,$$

which consists of all the unknown dynamics and the coupling terms.

In the ideal case, suppose the value of f_i can be obtained in real time, then the control input u_i could be designed as $u_i=\frac{1}{b_{Ei}}\left(-f_i+u_i^*\right)$. Hence, the system (4) becomes

$$y_i^{(n_i)}=u_i^*. \tag{5}$$

Moreover, suppose $y_i, y_i^{(1)}, \ldots y_i^{(n_i-1)}$ are all available, the ideal control law could be set

$$u_i^*=-\sum_{j=1}^{n_i} p_{ij} y_i^{(j-1)}, \tag{6}$$

where $p_{i1}, p_{i2}, \ldots p_{in_i}$ are the controller parameters. Then the channel i in the closed loop would be

$$y_i^{(n_i)}=-\sum_{j=1}^{n_i} p_{ij} y_i^{(j-1)}, \tag{7}$$

which can be assigned according to the practical requirements.

However, the total uncertainty f_i and the derivatives of y_i are usually unmeasurable in practice. Hence, to estimate these variables, the ESO can be designed for the channel i [9, 10]. In this paper, the following linear ESO is used:

$$\begin{cases} \dot{z}_{i\ 1}=z_{i\ 2}-\beta_{i\ 1}(z_{i\ 1}-y_i) \\ \dot{z}_{i\ 2}=z_{i\ 3}-\beta_{i\ 2}(z_{i\ 1}-y_i) \\ \cdots \\ \dot{z}_{i\ j}=z_{i\ j+1}-\beta_{i\ j}(z_{i\ 1}-y_i) \\ \cdots \\ \dot{z}_{i\ n_i+1-2}=z_{i\ n_i}-\beta_{i\ n_i-1}(z_{i\ 1}-y_i) \\ \dot{z}_{i\ n_i}=z_{i\ n_i+1}-\beta_{i\ n_i}(z_{i\ 1}-y_i)+b_{Ei}u_i \\ \dot{z}_{i\ n_i+1}=-\beta_{i\ n_i+1}(z_{i\ 1}-y_i) \end{cases} \tag{8}$$

where $b_{Ei}, \beta_{i\ 1}, \beta_{i\ 2} \ldots \beta_{i\ n_i+1}$ are the adjustable parameters.

If the command signals are given for the ith channel, then the control law can be designed as

$$u_i \hat{=} \frac{1}{b_{Ei}}\left(-z_{i\ n_i+1}+\sum_{j=1}^{n_i} p_{i\ j}\left(r_i^{(j-1)}-z_{i\ j}\right)\right) \tag{9}$$

where $p_{i\ 1}, p_{i\ 2}, \ldots p_{i\ n_i}$ are the controller parameters; $r_i^{(0)}=r_i(t)$ is the reference of the channel i; and $r_i^{(j)}$ $(j \geq 1)$ is the jth-order derivative of $r_i(t)$.

Consequently, the decentralized control scheme of ADRC is built as i runs from 1 to m. If the ESO works well, the presented ADRC design can stabilized the MIMO system without the accurate information about the orders or the relative degrees.

Combining the plant model and the controller scheme, the ADRC closed-loop system is obtained:

$$\vec{Y}(s)=G_M(s)\vec{R}(s) \tag{10}$$

where $\vec{R}=[R_1, R_2, \ldots R_m]^T$ are the Laplace transformations of the reference signal vector, $G_M(s)=A(s)(sH_D(s)A(s)+H_N(s)B_Y(s))^{-1}H_N(s)B_R(s)A^{-1}(s)$,

$$A(s)=\mathrm{diag}(b_{E1}a_1(s), \ldots b_{Em}a_m(s)),$$
$$a_i(s)=\sum_{k=1}^{n_i+1}\left(\sum_{j=k}^{n_i+1} p_{ij}\beta_{i\ j-k}\right)s^{k-1}, p_{i\ n_i+1}=1, \beta_{i\ 0}=1,$$
$$B_Y(s)=\mathrm{diag}(b_{Y1}(s), \ldots b_{YM}(s)), b_{Yi}(s)=\sum_{j=0}^{n_i} s^j \sum_{k=1}^{j+1} \beta_{i\ n_i+k-j} p_{i\ k},$$
$$B_R(s)=\mathrm{diag}(b_{R1}(s), \ldots b_{RM}(s)), b_{Ri}(s)=\left(\sum_{j=0}^{n_i+1} \beta_{i\ n-j} s^j\right)\left(\sum_{k=1}^{n_i} p_{i\ k} s^{k-1}\right).$$

Remark 2 Since $H_N(0)$ and $B_Y(0)=B_R(0)$ are both invertible, there is

$$G_M(0)=A(0)(H_N(0)B_Y(0))^{-1}H_N(0)B_R(0)A^{-1}(0)=I,$$

where I represents the unit matrix. Hence, the zero-frequency gain of the closed-loop system (10) is the unit matrix.

Moreover, several plants are given in Table 1 for later use.

3 Necessary condition for the closed-loop stability

To make sure of the closed-loop stability, the tuning of b_{Ei} is very important for the low-order ADRC scheme. In the special case of $m=1$, the plant (1) is a SISO system, $B_P=b_{11}$ is scalar, and $b_{E1}b_{11}>0$ is necessary for the closed-loop stability [32, 33, 35, 36]. Hence, the sign of b_{E1} is uniquely determined by $b_{11}>0$ in the SISO case. The corresponding necessary result for the MIMO system is shown by the following theorem.

Table 1 Plants employed in the simulation examples

Plant name	Plant transfer function
H_0	$\begin{bmatrix} \frac{1}{s+2} & \frac{1}{s+2} \\ \frac{1}{s+2} & \frac{1}{s+2} \end{bmatrix}$
H'_0	$\begin{bmatrix} \frac{1.2}{s+2} & \frac{1.2}{s+2} \\ \frac{1.2}{s+2} & \frac{1.2}{s+2} \end{bmatrix}$
H_1	$\begin{bmatrix} \frac{12.8}{16.7s+1} & \frac{12.8}{16.7s+1} \\ \frac{12.8}{16.7s+1} & \frac{12.8}{16.7s+1} \end{bmatrix}$
H'_1	$\begin{bmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{12.8e^{-s}}{16.7s+1} \\ \frac{12.8e^{-s}}{16.7s+1} & \frac{12.8e^{-s}}{16.7s+1} \end{bmatrix}$
H_2	$\begin{bmatrix} \frac{-2.2}{7s+1} & \frac{-2.2}{7s+1} \\ \frac{-2.2}{7s+1} & \frac{-2.2}{7s+1} \end{bmatrix}$
H'_2	$\begin{bmatrix} \frac{-2.2e^{-s}}{7s+1} & \frac{-2.2e^{-s}}{7s+1} \\ \frac{-2.2e^{-s}}{7s+1} & \frac{-2.2e^{-s}}{7s+1} \end{bmatrix}$
H_3	$\begin{bmatrix} \frac{0.126}{60s+1} & \frac{0.126}{60s+1} \\ \frac{0.126}{60s+1} & \frac{0.126}{60s+1} \end{bmatrix}$
H'_3	$\begin{bmatrix} \frac{0.126e^{-6s}}{60s+1} & \frac{0.126e^{-6s}}{60s+1} \\ \frac{0.126e^{-6s}}{60s+1} & \frac{0.126e^{-6s}}{60s+1} \end{bmatrix}$
H_4	$\begin{bmatrix} \frac{22.89}{4.572s+1} & \frac{22.89}{4.572s+1} \\ \frac{22.89}{4.572s+1} & \frac{22.89}{4.572s+1} \end{bmatrix}$
H'_4	$\begin{bmatrix} \frac{22.89e^{-0.2s}}{4.572s+1} & \frac{22.89e^{-0.2s}}{4.572s+1} \\ \frac{22.89e^{-0.2s}}{4.572s+1} & \frac{22.89e^{-0.2s}}{4.572s+1} \end{bmatrix}$
H_5	$\begin{bmatrix} \frac{0.66}{6.7s+1} & \frac{0.66}{6.7s+1} & \frac{0.66}{6.7s+1} \\ \frac{0.66}{6.7s+1} & \frac{0.66}{6.7s+1} & \frac{0.66}{6.7s+1} \\ \frac{0.66}{6.7s+1} & \frac{0.66}{6.7s+1} & \frac{0.66}{6.7s+1} \end{bmatrix}$

Table 1 (continued)

Plant name	Plant transfer function
H'_5	$\begin{bmatrix} \frac{0.66e^{-2.6s}}{6.7s+1} & \frac{0.66e^{-2.6s}}{6.7s+1} & \frac{0.66e^{-2.6s}}{6.7s+1} \\ \frac{0.66e^{-2.6s}}{6.7s+1} & \frac{0.66e^{-2.6s}}{6.7s+1} & \frac{0.66e^{-2.6s}}{6.7s+1} \\ \frac{0.66e^{-2.6s}}{6.7s+1} & \frac{0.66e^{-2.6s}}{6.7s+1} & \frac{0.66e^{-2.6s}}{6.7s+1} \end{bmatrix}$
H_6	$\begin{bmatrix} \frac{2.22}{(36s+1)(25s+1)} & \frac{2.22}{(36s+1)(25s+1)} & \frac{2.22}{(36s+1)(25s+1)} & \frac{2.22}{(36s+1)(25s+1)} \\ \frac{2.22}{(36s+1)(25s+1)} & \frac{2.22}{(36s+1)(25s+1)} & \frac{2.22}{(36s+1)(25s+1)} & \frac{2.22}{(36s+1)(25s+1)} \\ \frac{2.22}{(36s+1)(25s+1)} & \frac{2.22}{(36s+1)(25s+1)} & \frac{2.22}{(36s+1)(25s+1)} & \frac{2.22}{(36s+1)(25s+1)} \\ \frac{2.22}{(36s+1)(25s+1)} & \frac{2.22}{(36s+1)(25s+1)} & \frac{2.22}{(36s+1)(25s+1)} & \frac{2.22}{(36s+1)(25s+1)} \end{bmatrix}$
H'_6	$\begin{bmatrix} \frac{2.22e^{-2.5s}}{(36s+1)(25s+1)} & \frac{2.22e^{-2.5s}}{(36s+1)(25s+1)} & \frac{2.22e^{-2.5s}}{(36s+1)(25s+1)} & \frac{2.22e^{-2.5s}}{(36s+1)(25s+1)} \\ \frac{2.22e^{-2.5s}}{(36s+1)(25s+1)} & \frac{2.22e^{-2.5s}}{(36s+1)(25s+1)} & \frac{2.22e^{-2.5s}}{(36s+1)(25s+1)} & \frac{2.22e^{-2.5s}}{(36s+1)(25s+1)} \\ \frac{2.22e^{-2.5s}}{(36s+1)(25s+1)} & \frac{2.22e^{-2.5s}}{(36s+1)(25s+1)} & \frac{2.22e^{-2.5s}}{(36s+1)(25s+1)} & \frac{2.22e^{-2.5s}}{(36s+1)(25s+1)} \\ \frac{2.22e^{-2.5s}}{(36s+1)(25s+1)} & \frac{2.22e^{-2.5s}}{(36s+1)(25s+1)} & \frac{2.22e^{-2.5s}}{(36s+1)(25s+1)} & \frac{2.22e^{-2.5s}}{(36s+1)(25s+1)} \end{bmatrix}$

Theorem 1 A necessary condition for the stability of the closed loop formed by the plant (1) and the ADRC (7)(8) is

$$\det B_P \prod_{i=1}^{m} b_{Ei} > 0. \tag{11}$$

The proof is given in the appendix.

According to Theorem 1, once (11) is not satisfied, the ADRC closed loop will be unstable. Compared with the necessary condition for the SISO case, the condition in Theorem 1 is relaxed, even the sign of b_{Ei} for channel i is not uniquely determined for some systems. The plant H_0 in Table 1 is an example.

The following two kinds of parameters can be applied to the plantH_0:

$$b_{E1} = 1, b_{E2} = -5, n_i = 1, p_{i1} = 10, \beta_{i1} = 100, \beta_{i2} = 1000, i = 1, 2, \tag{12}$$

$$b_{E1} = -5, b_{E2} = 1, n_i = 1, p_{i1} = 10, \beta_{i1} = 100, \beta_{i2} = 1000, i = 1, 2. \tag{13}$$

The controlled outputs generated by (12) and (13) are illustrated by the solid black line and the dash-dotted blue line in Fig. 1, respectively. Both closed-loop systems are stable, although the sign of each b_{Ei} $(i = 1, 2)$ differs in (12) and (13). This phenomenon reveals an interesting difference between the MIMO systems and the SISO systems, which means the parameter b_{Ei} $(1 \leq i \leq m)$ for MIMO systems can be tuned in wider intervals.

4 Parameter tuning method and the sufficient condition for the closed-loop stability

From the experiments in the existing references (especially in [3, 28, 33]), an ADRC tuning method can be summarized, which is shown in the flowchart Fig. 2. The first step in the flowchart is to choose the order n_i for each channel, which is sets 1 or 2 in most of the ADRC applications. Then $\beta_{i\ 1}, \beta_{i\ 2} \ldots \beta_{i\ n_i+1}$ and $p_{i\ 1}, p_{i\ 2}, \ldots p_{i\ n_i}$ for $i = 1, \ldots, m$ are set according to their physical meanings and the design objects. Considering formula (7) and (8), $p_{i\ 1}, p_{i\ 2}, \ldots p_{i\ n_i}$ are determined by the expected closed-loop performances, $\beta_{i\ 1}, \beta_{i\ 2} \ldots \beta_{i\ n_i+1}$ should guarantee the convergence of the ESOs. For the initial value of b_{Ei} $(i = 1, \ldots, m)$, the sign (positive or

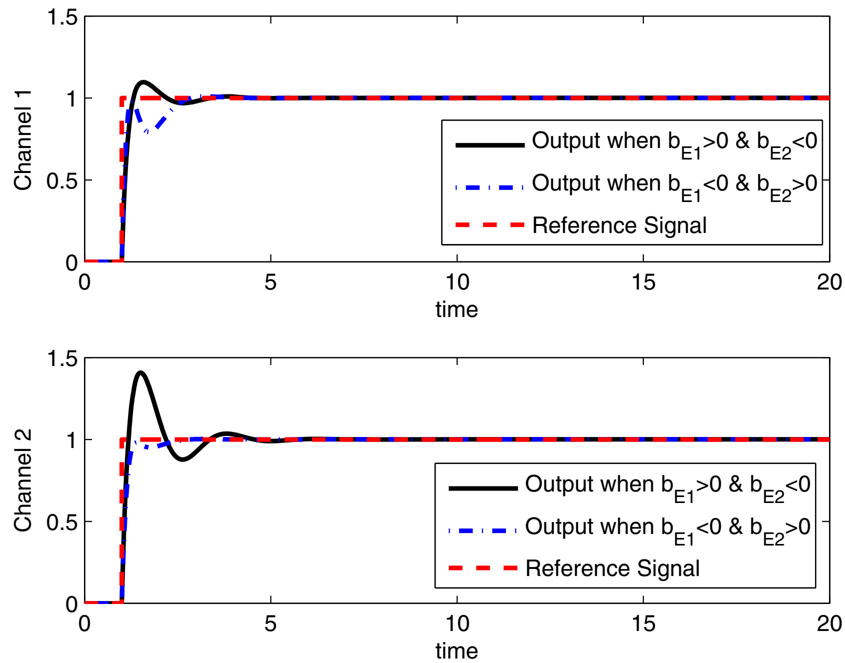


Fig. 1 Outputs of H_0 under ADRC with parameters (12) and (13)

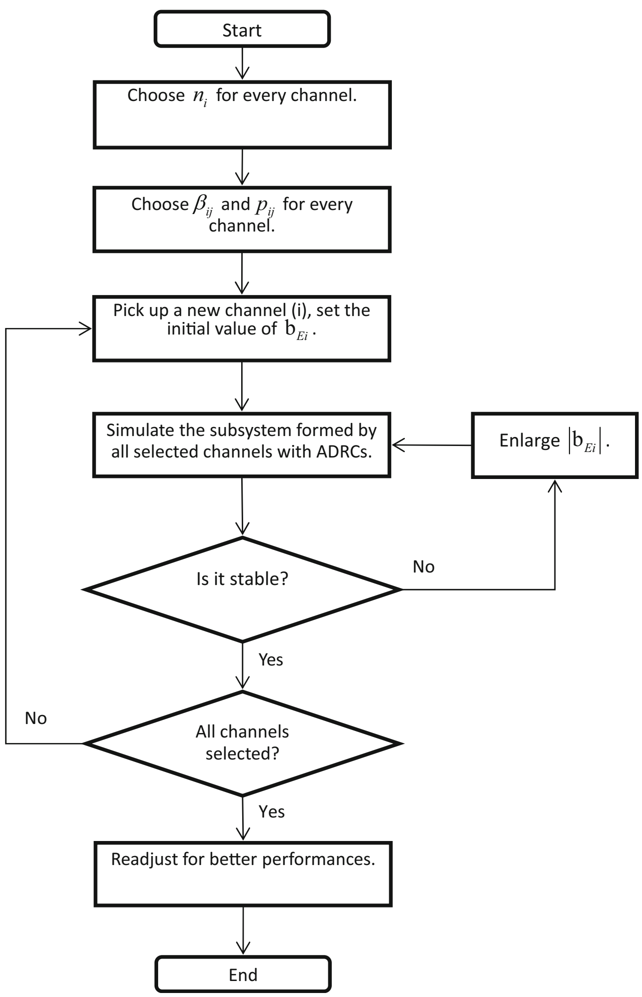


Fig. 2 Tuning method of the decentralized low-order ADRC scheme

negative) should be selected properly based on the engineering experiences.

In fact, the tuning method does not require the information of the plant order or the relative degree, and implies the sufficient condition of the closed-loop stability. Such mathematical law is given in the following theorem.

Theorem 2 For the closed-loop system formed by MIMO plant (1) and the group of ADRC (7)(8), assumes that $h_{Di}(s)$, $1 \leq i \leq m$ and $a_i(s)$, $1 \leq i \leq m$ are Hurwitz polynomials, $n_i \geq 1$ for $1 \leq i \leq m$, and

$$b_{Ei}\det B_{Pi-1}/\det B_{Pi} > 0, 1 \leq i \leq m \tag{14}$$

where $B_{P0} = 1$, and B_{Pi} is the sub-matrix of B_p formed by the intersection of the top i lines and the left i columns. Then the closed loop is stable when the absolute value of each b_{Ei}, $1 \leq i \leq m$ is big enough.

The proof is given in the appendix.

Remark 3 Since $\deg a_i(s) = n_i$ and all parameters of $a_i(s)$ are positive, $a_i(s)$ is a Hurwitz polynomial when $1 \leq n_i \leq 2$. Hence, for the low-order ADRC scheme, Theorem 2 can support for the tuning method in Fig. 2 theoretically.

This tuning method is applied to the six LTI systems $H_1(s)$, $H_2(s)$, $\ldots H_6(s)$, of which the transfer functions are listed in Table 1. Take $H_1(s)$ for example. Sets $n_i = 1$, $p_{i1} = 10$, $\beta_{i1} = 200$, and $\beta_{i2} = 1000$ for $i = 1, 2$. Then, sets $b_{E1} > 0$ and $b_{E2} < 0$ based on (14), and enlarge their absolute values until the closed loop is stable. The performance of the ADRC scheme with such parameters is illustrated in the first line of

Table 2 Decentralized ADRC with $n_i = 1$ for all channels

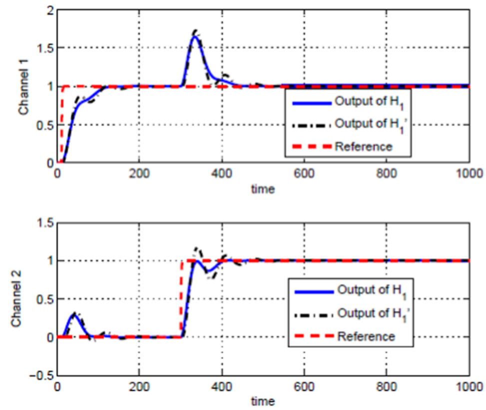
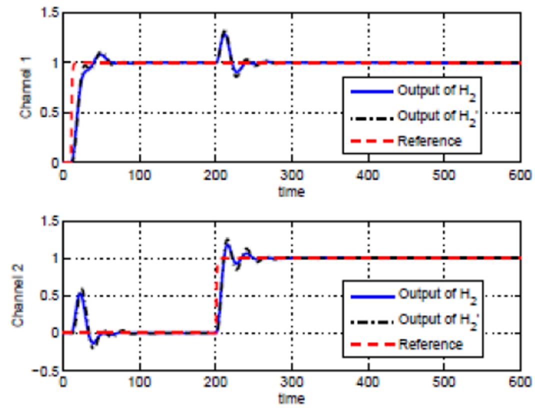
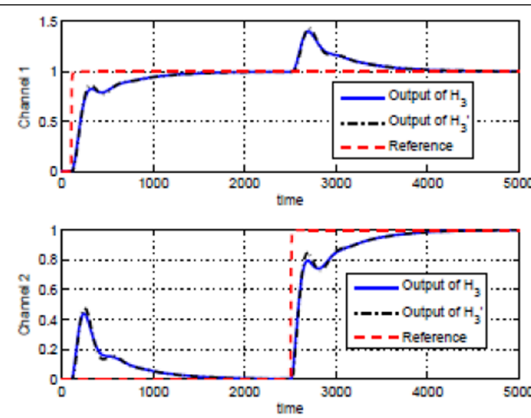
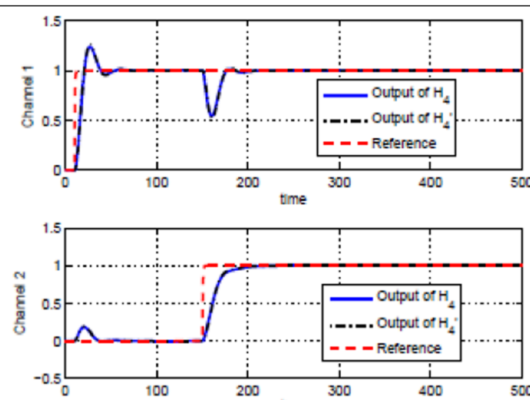
Plant	Controller parameter	Closed-Loop dynamic
H_1, H'_1	$b_{E1} = 10000,$ $\beta_{11} = 200, \beta_{12} = 1000,$ $p_{11} = 10,$ $b_{E2} = -10000,$ $\beta_{21} = 200, \beta_{22} = 1000,$ $p_{21} = 10.$	
H_2, H'_2	$b_{E1} = -500,$ $\beta_{11} = 200, \beta_{12} = 1000,$ $p_{11} = 10,$ $b_{E2} = 500,$ $\beta_{21} = 200, \beta_{22} = 1000,$ $p_{21} = 10.$	
H_3, H'_3	$b_{E1} = 500,$ $\beta_{11} = 200, \beta_{12} = 1000,$ $p_{11} = 10,$ $b_{E2} = -500,$ $\beta_{21} = 200, \beta_{22} = 1000,$ $p_{21} = 10.$	
H_4, H'_4	$b_{E1} = 5000,$ $\beta_{11} = 200, \beta_{12} = 1000,$ $p_{11} = 10,$ $b_{E2} = 5000,$ $\beta_{21} = 200, \beta_{22} = 1000,$ $p_{21} = 10.$	

Table 2 (continued)

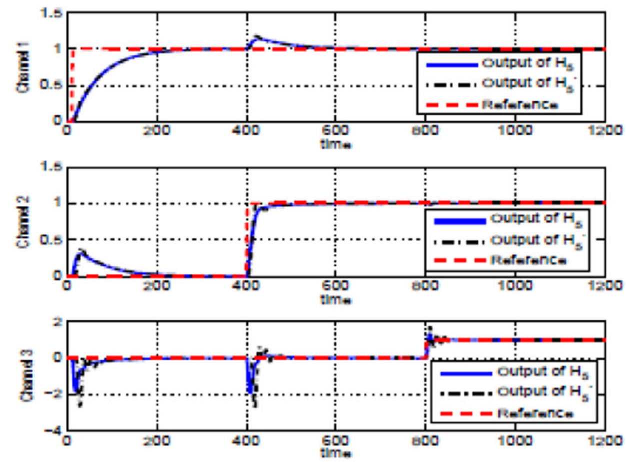
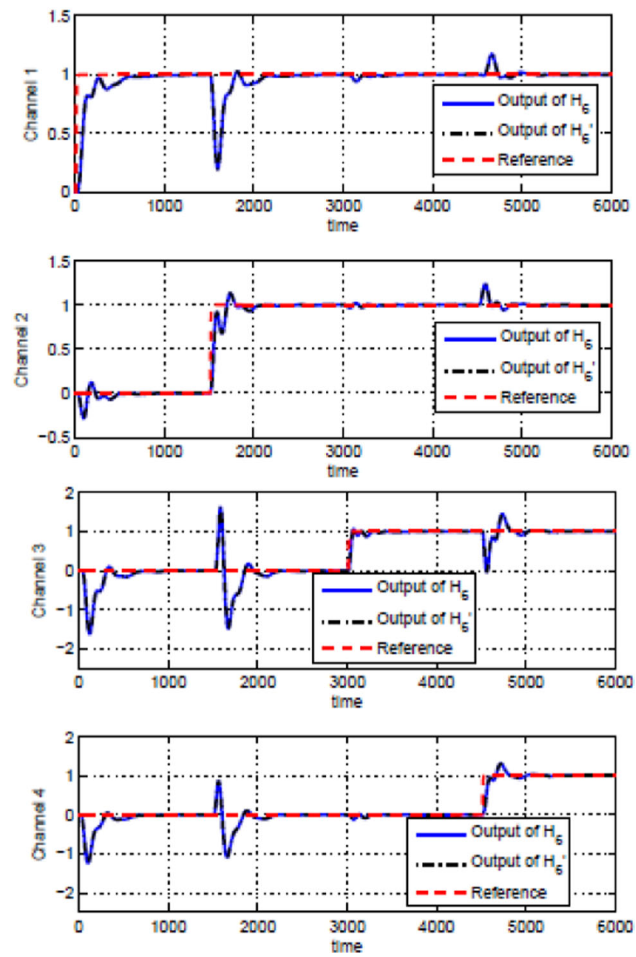
H_5, H'_5	$b_{E1}=1000$, $\beta_{11}=200$, $\beta_{12}=1000$, $p_{11}=10$, $b_{E2}=-1000$, $\beta_{21}=200$, $\beta_{22}=1000$, $p_{21}=10$, $b_{E3}=500$, $\beta_{31}=200$, $\beta_{32}=1000$, $p_{31}=300$.	
H_6, H'_6	$b_{E1}=15000$, $\beta_{11}=500$, $\beta_{12}=10000$, $p_{11}=10$, $b_{E2}=15000$, $\beta_{21}=500$, $\beta_{22}=10000$, $p_{21}=10$, $b_{E3}=15000$, $\beta_{31}=500$, $\beta_{32}=10000$, $p_{31}=10$, $b_{E4}=15000$, $\beta_{41}=500$, $\beta_{42}=10000$, $p_{41}=10$.	

Table 2. In the simulation, the two outputs of $H_1(s)$ are controlled to follow two step signals with different step times. The ADRC design procedures are similar for the six plants according to Fig. 2. Also, the controller parameters and simulation results are given in Table 2.

Since many high-order industrial processes are substituted by the models of reduced order with time delay in simulations [37], further study is made for the system with the time-delay term. The models with the time delay are absorbed from [31], including the Wood-Berry model (H_1'), the Vinante-Luyben

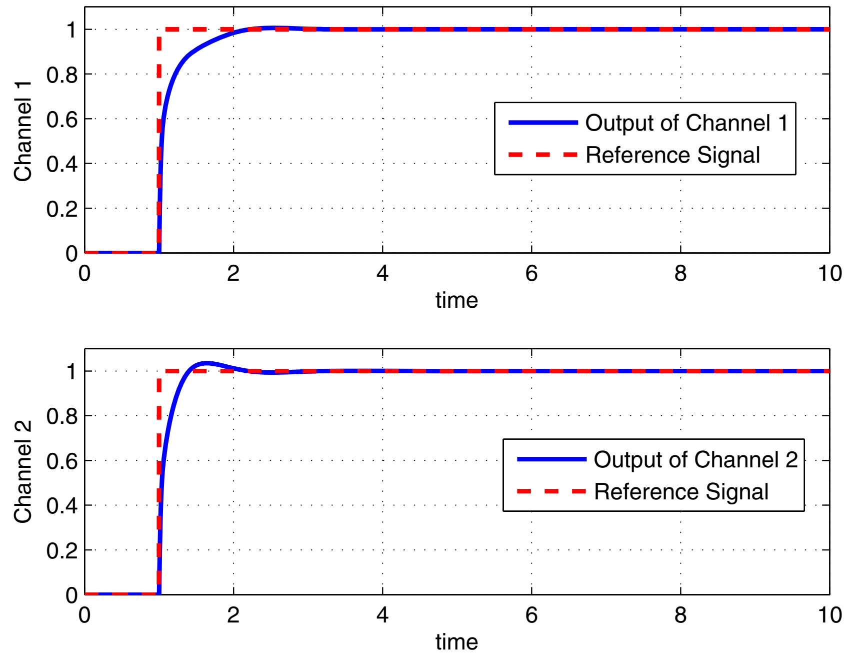


Fig. 3 Outputs of H_0' under ADRC with parameters in (16)

model (H_2'), the Wardle-Wood model (H_3'), the Ogunnaike-Ray model with two inputs and two outputs (H_4'), the Ogunnaike-Ray model with three inputs and three outputs (H_5'), and the Alatiqi case (H_6'). They are also listed in Table 1, and all built for the industrial processes. As shown in Table 1, $H_1'(s)$, $H_2'(s), \ldots$ $H_6'(s)$ can be obtained by adding the proper time-delay terms to $H_1(s)$, $H_2(s)$, … $H_6(s)$, respectively. And the same ADRC schemes with the same parameters with H_i, $1 \le i \le 6$ are applied to H_i', $1 \le i \le 6$ correspondingly. The simulation results are given in Table 2. Although the performances are lowered since the existence of the time delay, the closed loops are still stable.

5 Integrity of the closed-loop system

The integrity means that the closed-loop system remains stable when some channels are turned off [3]. Hence, it is about

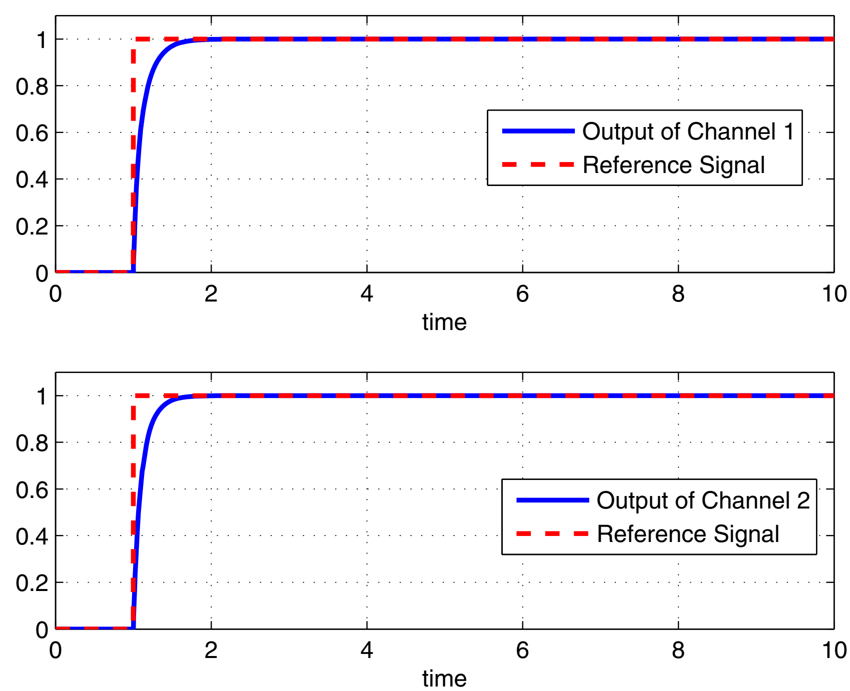


Fig. 4 Single channel outputs of H_0' controlled by ADRC scheme with parameters in (16) when the other channel is turned off

Table 3 Integrity test for H6 with the ADRC scheme

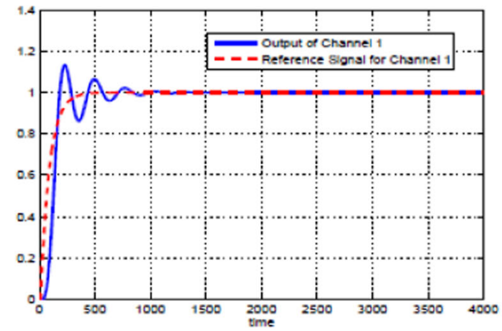
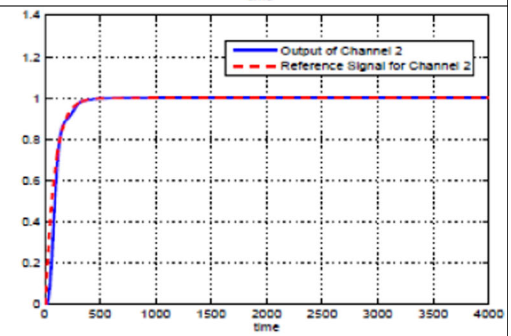
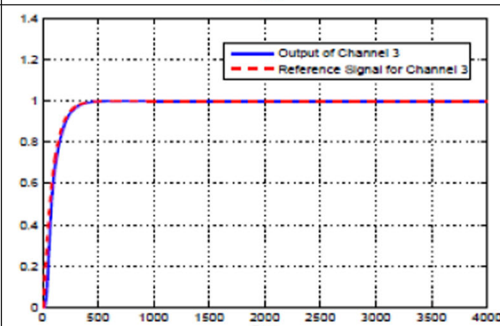
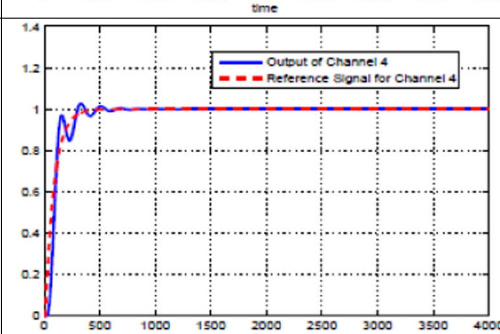
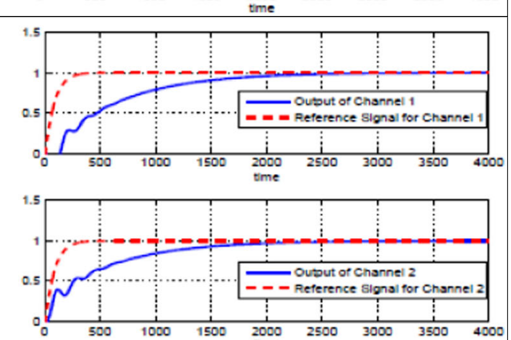
Equivalent Residue Model	Closed-Loop Dynamic
$y_1 = \frac{2.22}{(36s+1)(25s+1)} u_1$	
$y_2 = \frac{3.46}{32s+1} u_2$	
$y_3 = \frac{4.41}{16.2s+1} u_3$	
$y_4 = \frac{4.78}{(48s+1)(5s+1)} u_4$	
$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \frac{2.22}{(36s+1)(25s+1)} & \frac{-2.94(7.9s+1)}{(23.7s+1)^2} \\ \frac{-2.33}{(35s+1)^2} & \frac{3.46}{32s+1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$	

Table 3 (continued)

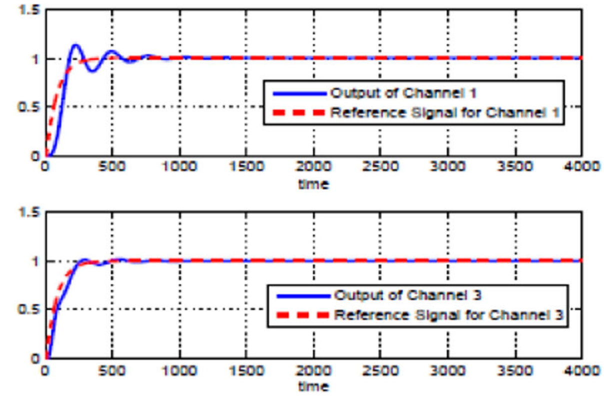
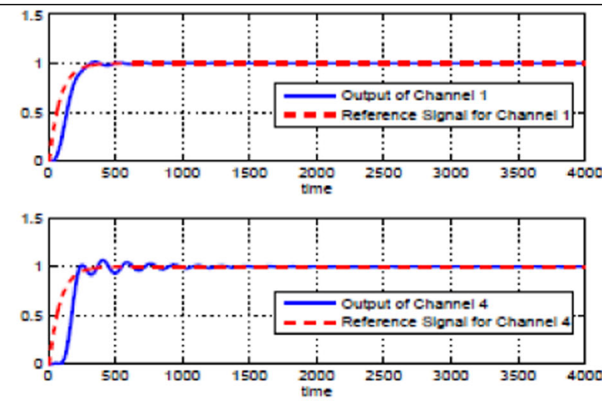
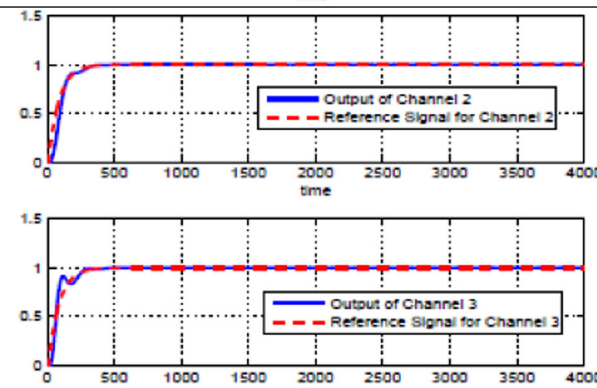
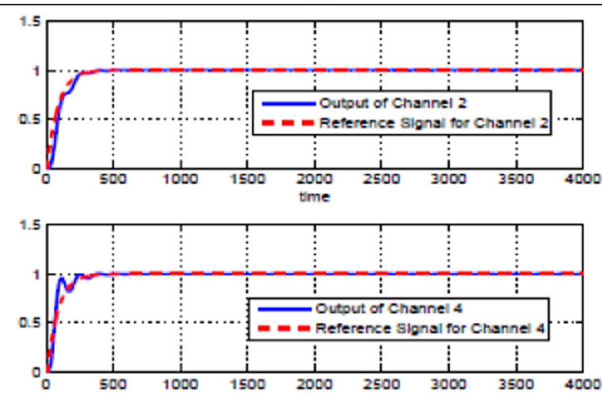
$\begin{bmatrix} y_1 \\ y_3 \end{bmatrix} = \begin{bmatrix} \frac{2.22}{(36s+1)(25s+1)} & \frac{0.017}{(31.6s+1)(7s+1)} \\ \frac{-1.06}{(17s+1)^2} & \frac{4.41}{16.2s+1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_3 \end{bmatrix}$	
$\begin{bmatrix} y_1 \\ y_4 \end{bmatrix} = \begin{bmatrix} \frac{2.22}{(36s+1)(25s+1)} & \frac{-0.64}{(29s+1)^2} \\ \frac{-5.73}{(8s+1)(50s+1)} & \frac{4.78}{(48s+1)(5s+1)} \end{bmatrix} \begin{bmatrix} u_1 \\ u_4 \end{bmatrix}$	
$\begin{bmatrix} y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} \frac{3.46}{32s+1} & \frac{-0.51}{(32s+1)^2} \\ \frac{3.511}{(12s+1)^2} & \frac{4.41}{16.2s+1} \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix}$	
$\begin{bmatrix} y_2 \\ y_4 \end{bmatrix} = \begin{bmatrix} \frac{3.46}{32s+1} & \frac{1.68}{(28s+1)^2} \\ \frac{4.32(25s+1)}{(50s+1)(5s+1)} & \frac{4.78}{(48s+1)(5s+1)} \end{bmatrix} \begin{bmatrix} u_2 \\ u_4 \end{bmatrix}$	

Table 3 (continued)

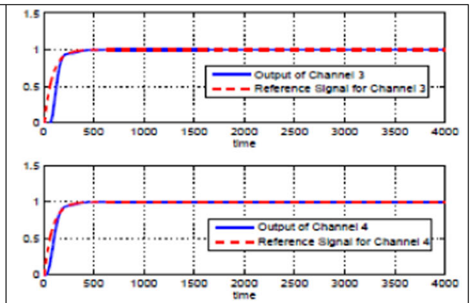
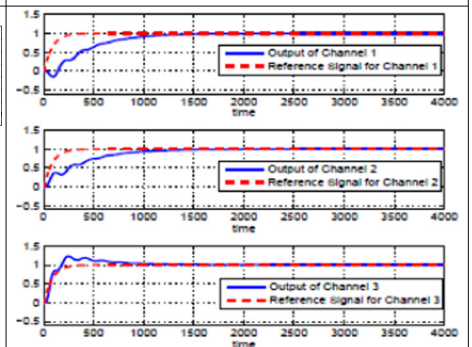
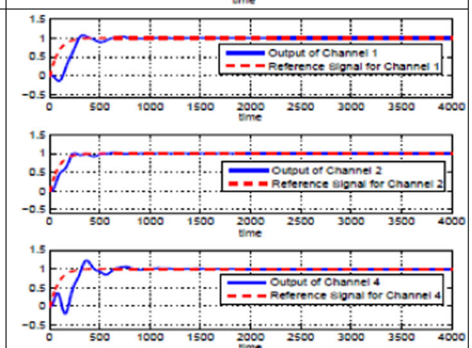
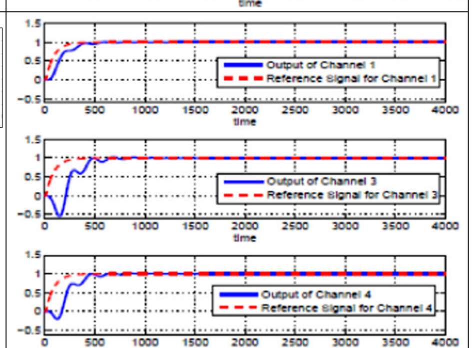
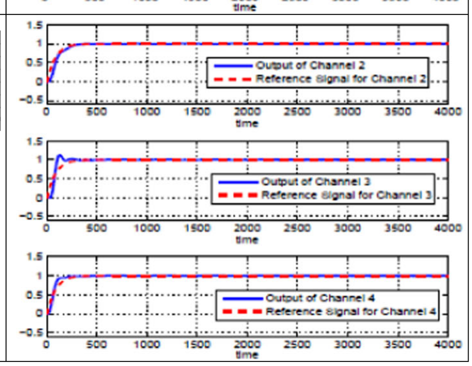
$\begin{bmatrix} y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} \frac{4.41}{16.2s+1} & \frac{-5.38}{17s+1} \\ \frac{-1.25}{(43.6s+1)(9s+1)} & \frac{4.78}{(48s+1)(5s+1)} \end{bmatrix} \begin{bmatrix} u_3 \\ u_4 \end{bmatrix}$	
$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} \frac{2.22}{(36s+1)(25s+1)} & \frac{-2.94(7.9s+1)}{(23.7s+1)^2} & \frac{0.017}{(31.6s+1)(7s+1)} \\ \frac{-2.33}{(35s+1)^2} & \frac{3.46}{32s+1} & \frac{-0.51}{(32s+1)^2} \\ \frac{-1.06}{(17s+1)^2} & \frac{3.511}{(12s+1)^2} & \frac{4.41}{16.2s+1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$	
$\begin{bmatrix} y_1 \\ y_2 \\ y_4 \end{bmatrix} = \begin{bmatrix} \frac{2.22}{(36s+1)(25s+1)} & \frac{-2.94(7.9s+1)}{(23.7s+1)^2} & \frac{-0.64}{(29s+1)^2} \\ \frac{-2.33}{(35s+1)^2} & \frac{3.46}{32s+1} & \frac{1.68}{(28s+1)^2} \\ \frac{-5.73}{(8s+1)(50s+1)} & \frac{4.32(25s+1)}{(50s+1)(5s+1)} & \frac{4.78}{(48s+1)(5s+1)} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_4 \end{bmatrix}$	
$\begin{bmatrix} y_1 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} \frac{2.22}{(36s+1)(25s+1)} & \frac{0.017}{(31.6s+1)(7s+1)} & \frac{-0.64}{(29s+1)^2} \\ \frac{-1.06}{(17s+1)^2} & \frac{4.41}{16.2s+1} & \frac{-5.38}{17s+1} \\ \frac{-5.73}{(8s+1)(50s+1)} & \frac{-1.25}{(43.6s+1)(9s+1)} & \frac{4.78}{(48s+1)(5s+1)} \end{bmatrix} \begin{bmatrix} u_1 \\ u_3 \\ u_4 \end{bmatrix}$	
$\begin{bmatrix} y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} \frac{3.46}{32s+1} & \frac{-0.51}{(32s+1)^2} & \frac{1.68}{(28s+1)^2} \\ \frac{3.511}{(12s+1)^2} & \frac{4.41}{16.2s+1} & \frac{-5.38}{17s+1} \\ \frac{4.32(25s+1)}{(50s+1)(5s+1)} & \frac{-1.25}{(43.6s+1)(9s+1)} & \frac{4.78}{(48s+1)(5s+1)} \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \\ u_4 \end{bmatrix}$	

Table 4 Decentralized ADRC with $n_i = 2$ for all channels

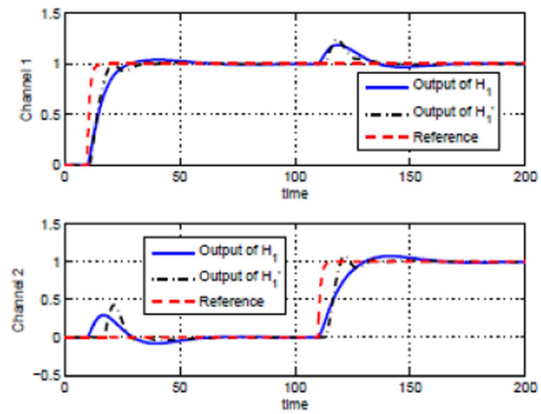
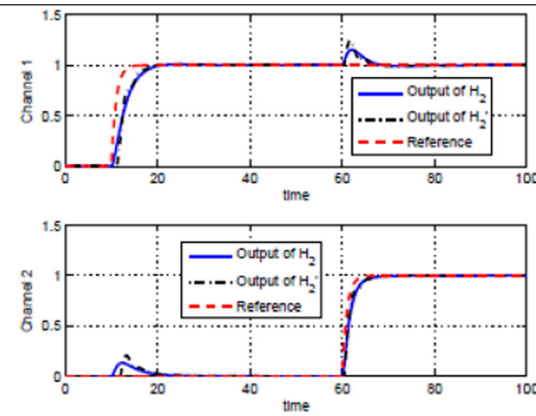
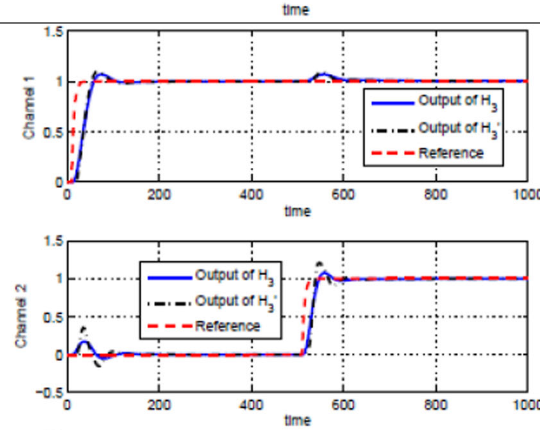
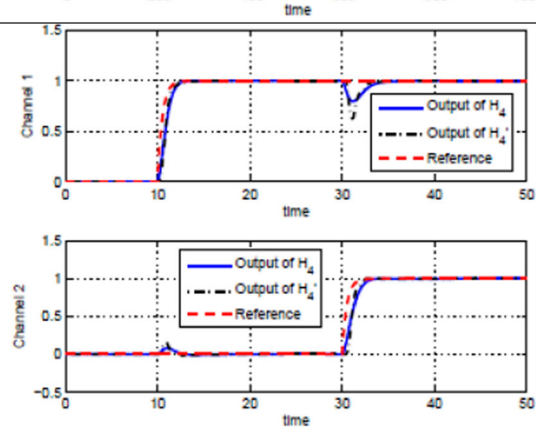
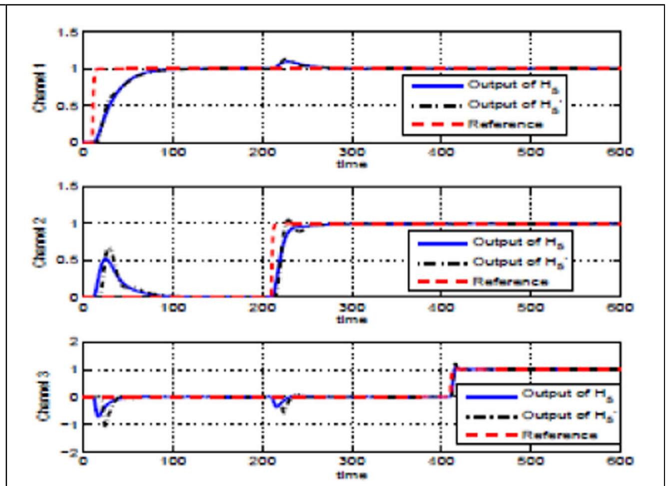
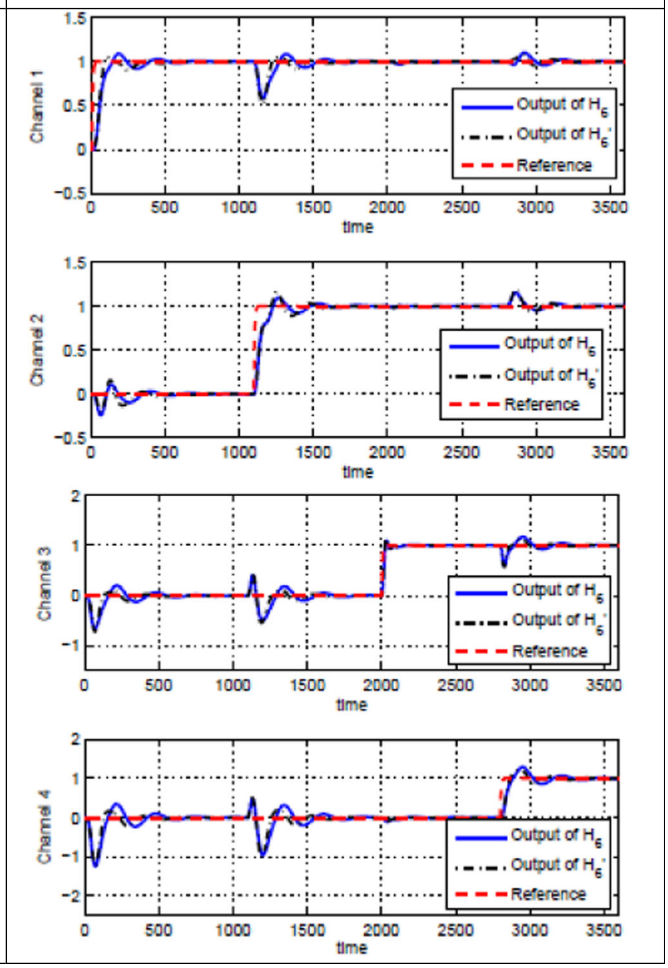
Plant	Controller Parameter	Closed-Loop Dynamic
H_1, H'_1	$b_{E1} = 1.8,\ \beta_{11} = 12,$ $\beta_{12} = 48,\ \beta_{13} = 8,$ $p_{11} = 1,\ p_{12} = 1.5,$ $b_{E2} = -5.6,\ \beta_{21} = 12,$ $\beta_{22} = 48,\ \beta_{23} = 10,$ $p_{21} = 1,\ p_{22} = 1.5.$	
H_2, H'_2	$b_{E1} = -11,\ \beta_{11} = 80,$ $\beta_{12} = 2200,\ \beta_{13} = 370,$ $p_{11} = 36,\ p_{12} = 8,$ $b_{E2} = 12,\ \beta_{21} = 1200,$ $\beta_{22} = 4800,\ \beta_{23} = 530,$ $p_{21} = 80,\ p_{22} = 12.$	
H_3, H'_3	$b_{E1} = 0.01,\ \beta_{11} = 4,$ $\beta_{12} = 5.5,\ \beta_{13} = 22,$ $p_{11} = 0.01,\ p_{12} = 0.14,$ $b_{E2} = -0.02,\ \beta_{21} = 2.5,$ $\beta_{22} = 2,\ \beta_{23} = 10.5,$ $p_{21} = 0.018,\ p_{22} = 0.18.$	
H_4, H'_4	$b_{E1} = 67,\ \beta_{11} = 230,$ $\beta_{12} = 18000,\ \beta_{13} = 4500,$ $p_{11} = 42,\ p_{12} = 9,$ $b_{E2} = 65,\ \beta_{21} = 290,$ $\beta_{22} = 28000,\ \beta_{23} = 20000,$ $p_{21} = 28,\ p_{22} = 7.$	

Table 4 (continued)

H_5, H'_5	$b_{E1}=13, \beta_{11}=2.9, \beta_{12}=2.7, \beta_{13}=135, p_{11}=0.0625, p_{12}=0.35, b_{E2}=-140, \beta_{21}=90, \beta_{22}=2700, \beta_{23}=35100, p_{21}=0.9, p_{22}=1.3, b_{E3}=10, \beta_{31}=210, \beta_{32}=15000, \beta_{33}=810000, p_{31}=1.21, p_{32}=1.52.$	
H_6, H'_6	$b_{E1}=1, \beta_{11}=156, \beta_{12}=8100, \beta_{13}=53000, p_{11}=0.0064, p_{12}=0.11, b_{E2}=5, \beta_{21}=264, \beta_{22}=23000, \beta_{23}=81000, p_{21}=0.032, p_{22}=0.25, b_{E3}=5, \beta_{31}=6, \beta_{32}=12, \beta_{33}=48, p_{31}=0.11, p_{32}=46, b_{E4}=1, \beta_{41}=180, \beta_{42}=10800, \beta_{43}=64800, p_{41}=0.025, p_{42}=0.21.$	

the stability of the subsystem of the closed loop. Since the MIMO system usually has more sensors and actuators than the SISO system, and the channels can be stopped by the hardware failure or other reasons, the integrity is important when designing the controller for the MIMO system.

In general, not all the open-loop stable MIMO plants can achieve the integrity with the decentralized ADRC scheme. For example, the integrity cannot be obtained for the system H_0 based on the ADRC. Since the determinant of the NZC matrix of H_0 is negative, $b_{E1}b_{E2}$ has to be negative according to Theorem 1. Once the channel i with $b_{Ei}>0$ breaks down

($i = 1$ or 2), the rest subsystem is formed by a SISO plant with a positive NZC and an ADRC with $b_{Ej} < 0,\ j \neq i$, which is unstable based on the SISO case of Theorem 1.

Consequently, the criterions for the integrity are necessary. The following theorem is established as a sufficient condition for the high integrity, which means the closed-loop system is stable when arbitrary many control signals are switched off.

Theorem 3 For the closed-loop system formed by MIMO plant (1) and the ADRC scheme (7)(8), it is assume that $h_{Di}(s),\ 1 \leq i \leq m$ and $a_i(s),\ 1 \leq i \leq m$ are Hurwitz polynomials, $n_i \geq 1$ for $1 \leq i \leq m$, and B_p is a diagonally dominant matrix, that is, $|b_{ii}| \geq \sum_{1 \leq j \leq m,\ \ j \neq i} |b_{ij}|$ holds for $1 \leq i \leq m$. Then there exists a group of controller parameters $b_{Ei}, p_{i\ 1}, p_{i\ 2}, \cdots p_{i\ n_i}, \beta_{i\ 1}, \beta_{i\ 2} \ldots \beta_{i\ n_i+1}, i = 1, \ldots, m$, satisfying

$$b_{Ei} b_{ii} > 0,\ \ 1 \leq i \leq m, \tag{15}$$

such that the closed-loop system is stable and has high-integrity property, which means the closed loop remains stable when arbitrary many (but not more than $m - 1$) $u_i(t)$ are forced to be zero. The proof is given in the appendix.

In [3], the decentralized ADRC scheme is built to keep the closed-loop stability when arbitrary $m - 1$ loops break down for several kinds of MIMO systems. And the Theorem 3 implies that the closed-loop stability can be guaranteed by the decentralized ADRC scheme with the fixed parameters when many arbitrary loops break down, provided that the NZC matrix of the plant is diagonally dominant. In fact, redefine the correspondence of the inputs and outputs for the system H_0, the plant $H_0^{'}$ in Table 1 is obtained, and $H_0^{'}$ has a diagonally dominant NZC matrix. Hence, the integrity can be obtained by the decentralized ADRC scheme with the following parameters:

$$\begin{aligned} b_{E1} = 1, b_{E2} = 1, n_i = 2, p_{i1} = 10, \beta_{i1} = 100, \beta_{i2} \\ = 1000, i = 1, 2. \end{aligned} \tag{16}$$

The controller performances are given in Figs. 3 and 4. In detail, Fig. 3 illustrates the outputs when the two channels of the system $H_0^{'}$ are available, and Fig. 4 shows the output of channel 1 (or 2) when channel 2 (or 1) is turned off. It can be seen that the closed-loop stability is obtained in all the circumstances, and the closed-loop performances are similar no matter the full channels are available or not.

Although Theorem 3 is proved for the plant with the diagonally dominant NZC matrix, the high-integrity property can be obtained for more kinds of systems. That means the corresponding closed loop is stable under any possible loop failures. The plant H_6 with four inputs and four outputs is taken as an example. And the ADRC scheme is the same as given in Table 2. When several channels of H_6 stop operating, the transfer function model of H_6 becomes a new model with less inputs and outputs, which is called the equivalent residue model (ERM) in this paper. Table 3 lists all the possible ERMs of H_6 and the corresponding closed-loop performances. It can be seen that the system is stable in all the cases.

6 Further discussions

For the decentralized low-order ADRC scheme, the closed-loop stability and integrity can be obtained based on Theorem 2 and Theorem 3 for the LTI MIMO plants. If the time-delay terms of the plants are in a certain range, the ADRC design can still work well even with the same parameters. The former simulation experiments are all on the ADRC scheme with $n_i = 1,\ \ 1 \leq i \leq m$. In fact, the same design method can also be applied to the ADRC scheme with $n_i = 2,\ \ 1 \leq i \leq m$.

Hence, the corresponding simulations are made based on the ADRC scheme with $n_i = 2,\ \ 1 \leq i \leq m$, of which the results are given in Table 4, and part of the controller parameters are from the experiments in [3, 31]. Moreover, the closed loop stability still holds without readjusting the controller parameters for the time-delay systems.

7 Conclusion

This study focuses on the design method of the decentralized low-order ADRC scheme for the uncertain MIMO plants, of which the orders and the relative degrees are unknown. The corresponding design and tuning procedure is shown in the flowchart (Fig. 2) and the related simulations. In the theoretical discussion, the NZC matrix of the MIMO system is generalized from the SISO case. Based on the NZC matrix, Theorem 1 establishes a necessary condition for the closed-loop stability. Theorem 2 indicates that the ADRC scheme with $n_i \geq 1,\ 1 \leq i \leq m$ can stabilize the open-loop stable plants. According to Theorem 3, the former method is capable to guarantee the integrity for the open-loop stable plant with the diagonally dominant NZC matrix, which has applied to more general plants in simulations. Theorems support for the ADRC design method from the theoretical aspect. Moreover, the decentralized ADRC schemes are designed for several industrial processes in. Simulation results imply that the design method is efficient for the ADRCs based on the 2-order or 3-order ESOs.

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Proof of the Theorems

Proof of Theorem 1

It will be shown that (11) is necessary for all the denominators of the main diagonal elements in $G_M(s)$ to be Hurwitz.

Let $d_{Mij}(s)$ be the denominator polynomial of the element in the ith row and jth column of $G_M(s)$, and denote

$$d_M(s) = \det(sH_D(s)A(s) + H_N(s)B_Y(s)).$$

Then there is

$$\begin{aligned} d_{Mii}(s) &= \det(sH_D(s)A(s) + H_N(s)B_Y(s)) \\ &= d_M(s), \quad 1 \le i \le m. \end{aligned}$$

According to the definition of the determinant based on the Leibniz formula [38], $d_M(s)$ can be expressed as

$$d_M(s) = d_{HM}(s) + d_{LM}(s),$$

where $d_{HM}(s) = \prod_{i=1}^{m} (b_{Ei}sa_i(s)h_{Di}(s) + b_{ii}b_{Yi}(s)h_{Nii})$ .. Since $\deg b_{Yi}(s) = n_i$, $\deg a_i(s) = n_i$, and $\deg h_{Nij}(s) = m_{ij} \le m_i = \deg h_{Di}(s)$ for $1 \le i \le m$, $1 \le j \le m$, there are

$$\deg d_{LM}(s) < \deg d_{HM}(s) = \deg d_M(s),$$
$$\deg b_{Ei}sa_i(s)h_{Di}(s) < \deg b_{ii}b_{Yi}(s)h_{Nii}.$$

Hence, the coefficients of the highest-order and the lowest-order terms in $d_M(s)$ are $\prod_{i=1}^{m} b_{Ei}$ and $d_M(0) = \det B_P$, respectively. So, if $d_M(s)$ is Hurwitz, then (11) holds.

Proof of Theorem 2

To prove Theorem 2, the following Lemma is needed:

Lemma 1 Assume $P(s)$ and $Q(s)$ are both polynomials with real coefficients, $P(s)$ is Hurwitz with $degP(s) = n$, $P(0)Q(0) > 0$, and $degQ(s) \le n + 1$. Then there exists some positive κ_0 such that $\kappa sP(s) + Q(s)$ is Hurwitz for any $\kappa > \kappa_0$.

Proof of Lemma 1 Without loss of generality, assume that $P(0) > 0$ and $Q(0) > 0$. Since $P(s)$ is Hurwitz, all the coefficients of $P(s)$ are positive. And, it is enough to prove that all the roots of $sP(s) + \kappa^{-1}Q(s)$ are on the left half plane when $\kappa > \kappa_0$.

According to the continuous dependence of the roots on the polynomial's coefficients, the roots of $sP(s) + \kappa^{-1}Q(s)$ move towards the roots of $sP(s)$ correspondingly as κ increases. When n of the $n + 1$ roots of $sP(s) + \kappa^{-1}Q(s)$ are close enough to the roots of $P(s)$, they will be on the left half plane, and the real part of their product multiplied by $(-1)^n$ will be positive.

Consequently, if the $(n + 1)$th root of $sP(s) + \kappa^{-1}Q(s)$ is the conjugation of one among the first n roots, then the $(n + 1)$th root is also on the left half plane. Otherwise, if the $(n + 1)$th root of $sP(s) + \kappa^{-1}Q(s)$ is not a conjugation of any other root, it has to be a real number, and the product of the other n roots multiplied by $(-1)^n$ is real positive. Since $(sP(s) + Q(s))|_{s=0} = Q(0) > 0$, the $(n + 1)$th root of $sP(s) + \kappa^{-1}Q(s)$ is a negative real number according to the relationship between roots and coefficients. That completes the proof.

Proof of Theorem 2 It will be proved by induction. For $m = 1$, the plant (1) is a SISO system, the inequality (14) becomes $b_{E1}b_{P1} > 0$, and the statement holds based on Theorem 2 in [33].

Suppose the statement holds when m equals the integer $K \ge 1$. That is, when $n_i \ge 1$ for $i = 1, \ldots, K$, there exists a group of controller parameters $b_{Ei}, p_{i\ 1}, p_{i\ 2}, \ldots p_{i\ n_i}, \beta_{i\ 1}, \beta_{i\ 2} \cdots \beta_{i\ n_i+1}, i = 1, \ldots, K$, satisfying $b_{Ei} \det B_{Pi-1} / \det B_{Pi} > 0$, such that all the elements in $G_{MK}(s)$ are stable transfer functions. Hence, $d_{MK}(s)$ and $a_i(s)$, $1 \le i \le K$ are all Hurwitz.

For $m = K + 1$, there is

$$\begin{aligned} & d_{K+1}(s) = \det(sH_D(s)A(s) + H_N(s)B_Y(s)) \\ = & \begin{vmatrix} b_{E1}sa_1h_{D1} + h_{N11}b_{Y1} & \cdots & h_{N1K}b_{YK} & h_{N1\ K+1}b_{Y\ K+1} \\ \vdots & \ddots & \vdots & \vdots \\ h_{NK1}b_{Y1} & \cdots & b_{EK}sa_Kh_{DK} + h_{NKK}b_{YK} & h_{N\ K\ K+1}b_{Y\ K+1} \\ h_{N\ K+1\ 1}b_{Y1} & \cdots & h_{N\ K+1\ K}b_{YK} & b_{E\ K+1}sa_{K+1}h_{D\ K+1} + h_{N\ K+1\ K+1}b_{Y\ K+1} \end{vmatrix} \\ = & \begin{vmatrix} b_{E1}sa_1h_{D1} + h_{N11}b_{Y1} & \cdots & h_{N1K}b_{YK} & h_{N1\ K+1}b_{Y\ K+1} \\ \vdots & \ddots & \vdots & \vdots \\ h_{NK1}b_{Y1} & \cdots & b_{EK}sa_Kh_{DK} + h_{NKK}b_{YK} & h_{N\ K\ K+1}b_{Y\ K+1} \\ 0 & \cdots & 0 & b_{E\ K+1}sa_{K+1}h_{D\ K+1} \end{vmatrix} \\ + & \begin{vmatrix} b_{E1}sa_1h_{D1} + h_{N11}b_{Y1} & \cdots & h_{N1K}b_{YK} & h_{N1\ K+1}b_{Y\ K+1} \\ \vdots & \ddots & \vdots & \vdots \\ h_{NK1}b_{Y1} & \cdots & b_{EK}sa_Kh_{DK} + h_{NKK}b_{YK} & h_{N\ K\ K+1}b_{Y\ K+1} \\ h_{N\ K+1\ 1}b_{Y1} & \cdots & h_{N\ K+1\ K}b_{YK} & h_{N\ K+1\ K+1}b_{Y\ K+1} \end{vmatrix} \\ = & b_{E\ K+1}sa_{K+1}(s)h_{D\ K+1}(s)d_{MK}(s) + b_{Y\ K+1}(s)q_{K+1}(s), \end{aligned}$$

where $q_{K+1}(s)$ is defined as

$$q_{K+1}(s)=\begin{vmatrix} b_{E1}sa_1h_{D1}+h_{N11}b_{Y1} & \cdots & h_{N1K}b_{YK} & h_{N1\ K+1} \\ \vdots & \ddots & \vdots & \vdots \\ h_{NK1}b_{Y1} & \cdots & b_{EK}sa_Kh_{DK}+h_{NKK}b_{YK} & h_{N\ K\ K+1} \\ h_{N\ K+1\ 1}b_{Y1} & \cdots & h_{N\ K+1\ K}b_{Y\ K} & h_{N\ K+1\ K+1} \end{vmatrix}.$$

According to the induction hypothesis, there exists a group of parameters $b_{Ei}, p_{i\ 1}, p_{i\ 2}, \ldots p_{i\ n_i}, \beta_{i\ 1}, \beta_{i\ 2} \ldots \beta_{i\ n_i+1}, i = 1, \ldots, K$, such that $d_{MK}(s)$ and $a_i(s)$, $1 \le i \le K$ are all Hurwitz. Then choose $p_{K+1\ 1}, p_{K+1\ 2}, \ldots p_{K+1\ n_{K+1}}, \beta_{K+1\ 1}, \beta_{K+1\ 2} \ldots \beta_{K+1\ n_{K+1}+1}$ such that $a_{K+1}(s)$ is Hurwitz. Hence, $a_{K+1}h_{D\ K+1}d_{MK}$ is a Hurwitz polynomial.

It can be verified by the definition of the determinant based on the Leibniz formula [38] that

$$\deg q_{K+1}(s) \le \deg d_{MK}(s) + \max_{1\le i\le K+1} \deg h_{N\ K+1\ i} \le \deg d_{MK}(s) + \deg h_{D\ K+1}.$$

Considering $\deg a_{K+1} = \deg b_{Y\ K+1}$, there is

$$\deg b_{Y\ K+1} q_{K+1} \le 1 + \deg a_{K+1} h_{D\ K+1} d_{MK}. \tag{17}$$

Then, according to Lemma 1, $d_{K+1} = b_{E\ K+1} s a_{K+1} h_{D\ K+1} d_{MK} + b_{Y\ K+1} q_{K+1}$ is Hurwitz when

$$d_{K+1}(0) \prod_{i=1}^{K+1} b_{Ei} > 0$$

and $|b_{E\ K+1}|$ is big enough.

Since $d_{K+1}(0) = \det B_p$ and $b_{Ei} \det B_{Pi-1} / \det B_{Pi} > 0$, $1 \le i \le K$, there is

$$b_{E\ K+1} \det B_{P\ K} / \det B_{P\ K+1} > 0.$$

Then, the induction is finished.

Considering $d_{Mij}(s) = a_j(s) d_M(s)$, $1 \le i \le m$, $1 \le j \le m$, all the denominators in $G_M(s)$ are Hurwitz when $d_M(s)$ and $a_j(s)$, $1 \le j \le m$ are Hurwitz. That completes the proof.

Proof of Theorem 3

To prove Theorem 3, the following Lemma is needed:

Lemma 2 Assume $P_i(s)$ and $Q_i(s)$, $1 \le i \le k$ are polynomials with real coefficients, $P_i(s)$, $1 \le i \le k$ are all Hurwitz, and $P_i(0)Q_i(0) > 0$, $degQ_i(s) \le degP_i(s) + 1$ for $1 \le i \le k$. Then there exists some positive κ_0 such that $\kappa s P_i(s) + Q_i(s)$, $1 \le i \le k$ are Hurwitz for any $\kappa > \kappa_0$.

Proof of Lemma 2 Apply Lemma 1 to $sP_i(s) + Q_i(s)$ for each i, then κ_{0i}, $1 \le i \le k$ are obtained. Let $\kappa = \max\ \{\kappa_{0i} | 1 \le i \le k\}$. The proof is finished.

Proof of Theorem 3 It will be proved by induction. For $m = 1$, the plant (1) is a SISO system, the inequality (15) becomes $b_{E1} b_{P1} > 0$. Hence, the closed-loop stability is obtained according to Theorem 2, and the integrity follows.

Suppose the statement holds when m equals the integer $K \ge 1$. That is, when $n_j \ge 1$ for $j = 1, \ldots, K$, there exists a group of controller parameters $b_{Ej}, p_{j\ 1}, p_{j\ 2}, \ldots p_{j\ n_j}, \beta_{j\ 1}, \beta_{j\ 2} \ldots \beta_{j\ n_j+1}, j = 1, \ldots, K$, satisfying $b_{Ej} b_{jj} > 0$, such that $G_{MK}(s; \mathbb{S})$ are stable for any possible $\mathbb{S}$, where $\mathbb{S} \not\subseteq \{1, \ldots, K\}$ is a set of the channel indexes, and $G_{MK}(s; \mathbb{S})$ denotes the closed-loop transfer function when the channels in S break down. And $d_{MK}(s; \mathbb{S})$, $q_K(s; \mathbb{S})$ can be defined similarly. Hence, $G_{MK}(s; \varnothing) = G_{MK}(s)$, $d_{MK}(s; \varnothing) = d_{MK}(s)$, $d_{MK}(s; \mathbb{S}), \mathbb{S} \not\subseteq \{1, \ldots, K\}$ and $a_j(s)$, $1 \le j \le K$ are all Hurwitz.

Suppose $m = K + 1$. If $K + 1 \in S$, there is

$$d_{K+1}(s; \mathbb{S}) = d_K(s; \mathbb{S}/\{K+1\}).$$

Then, according to the induction hypothesis, there exists a group of parameters $b_{Ej}, p_{j\ 1}, p_{j\ 2}, \ldots p_{j\ n_j}, \beta_{j\ 1}, \beta_{j\ 2} \ldots \beta_{j\ n_j+1}, j = 1, \ldots, K$, such that $d_{K+1}(s; \mathbb{S}), \mathbb{S} \not\subseteq \{1, \ldots, K\}$, and $a_j(s)$, $1 \le j \le K$ are all Hurwitz.

If $K + 1 \notin \mathbb{S}$, similar to the proof of Theorem 2, there is

$$d_{K+1}(s; \mathbb{S}) = b_{E\ K+1} s a_{K+1}(s) h_{D\ K+1}(s; \mathbb{S}) d_{MK}(s; \mathbb{S}) + b_{Y\ K+1}(s) q_{K+1}(s; \mathbb{S}).$$

Then choose $p_{K+1\ 1}, p_{K+1\ 2}, \ldots p_{K+1\ n_{K+1}}, \beta_{K+1\ 1}, \beta_{K+1\ 2} \ldots \beta_{K+1\ n_{K+1}+1}$ such that $a_{K+1}(s)$ is Hurwitz. Hence, $a_{K+1} h_{D\ K+1}(s; \mathbb{S}) d_{MK}(s; \mathbb{S})$ is a Hurwitz polynomial.

Similar to (17), there is

$$\deg b_{Y\ K+1}(s) q_{K+1}(s; \mathbb{S}) \le 1 + \deg a_{K+1}(s) h_{D\ K+1}(s; \mathbb{S}) d_{MK}(s; \mathbb{S}), \tag{18}$$

for any $\mathbb{S} \not\subseteq \{1, \ldots, K\}$. Thus, according to the Lemma 2, $d_{K+1}(s; \mathbb{S})$ is Hurwitz when

$$d_{K+1}(0) \prod_{j=1}^{K+1} b_{Ej} > 0 \tag{19}$$

and $|b_{E\ K+1}|$ is big enough. Considering B_p is diagonally dominant, $d_{K+1}(0; \mathbb{S}) = \det B_p(\mathbb{S})$ has the same sigh with $\prod_{1 \le j \le K+1,\ j \notin S} b_{jj}$. Since $b_{Ej} b_{jj} > 0$, $1 \le j \le K$, (19) is satisfied when

$$b_{E\ K+1} b_{K+1\ K+1} > 0.$$

Hence, the induction is completed.

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